Computerized Schedule Construction for a VTOL Airbus Transportation System

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As part of studies carried out for the U.S. Department of Transportation, a series of computer programs has been written which constructs a detailed time-of-day schedule for a hypothetical VTOL airbus system operating in the Northeast Corridor in the year 1980. Given basic demand data such as daily origin-destination passenger traffic, the frequency-of-service pattern for nonstop services is determined using network flow methods. A criterion of the number of daily, onboard passengers is used to qualify a route for nonstop service, and a desirable market load factor converts the passenger flows to a frequency pattern. Given information on the daily variations of demand, a range of desirable departure times is assigned to each proposed service, and an initial schedule is constructed by placing the departure time in the middle of its range. A heuristic program then optimizes the schedule, improving daily aircraft utilization by retiming the services so as to make better connections between nonstop services and to construct the multistop flights. The methods have been used to generate parametric schedules involving 100–200 aircraft serving 50 stops with over 3000 flights/day.

Introduction

A SCHEDULE plan is a complete description of a transportation system. It details the services to be offered in the dimensions of time and geography, gives the routing followed by vehicles, and indicates the loadings to be placed upon terminals. A complete statistical summary of the operation of the transportation system can be obtained once the schedule is completed. The number of vehicles and crews, their daily utilization, the expected load factors, the required number of loading gates, the average length of vehicle hop, etc. are all implicitly determined by the schedule plan. Constructing and maintaining an efficient schedule is the main problem of transportation system managements. It is both their production plan and their product to be marketed, and the economic success of the plan is gaged by the management's ability to produce a low-cost production that will be salable to the traveling public.

The use of computers in scheduling is not widespread at this time; and if they are used, it is generally for data processing as distinct from decision making or problem solving. The reasons for this are clear. There has not been in the past sufficient capability either in the hardware or the software to handle problems of the size and complexity associated with transportation systems. This situation has been changing in the last few years, to the point where we can now begin to handle fairly large-scale scheduling problems, introducing optimization at several points and constructing quickly and easily full system schedules and their statistical summaries. Parametric investigations of the effects of restricting fleet size, terminal size, etc. can then be quickly carried out. Various strategies or policy decisions are similarly easily investigated. For such questions, the computer methods become useful tools in the hands of a long-range planner of future transportation systems.

This report will describe briefly the methods that have been used for a hypothetical VTOL airbus short-haul system[1] in the Northeast Corridor. A more complete description of these methods is given in Ref. 2. It is only a beginning, as valuable extensions are yet to come. References 3–10 describe recent efforts at the Massachusetts Institute of Technology (MIT) in the areas of dynamic scheduling, crew scheduling, schedule control, and optimal dispatching. Other recent computer scheduling processes are discribed in Refs. 11 and 12.

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VTOL Airbus Schedule Construction

1. Basic Assumptions: *D* Matrix

Fifty stopping points representing centers of population within the Northeast Corridor area were arbitrarily selected. It was assumed that service was to be provided to all of these points by one VTOL system using one type of VTOL vehicle, an 80-passenger tilt-wing aircraft cruising at 350 knots. The schedule plan to be constructed was a fixed daily schedule for every day of some longer period, such as a month. The basic input demand data which could be forecast were the expected mean value for the period of the daily number of trips d_{ij}, originating at point i and terminating at point j. These data were contained in an O and D matrix called the D matrix.

This forecast demand has initially been assumed to be an independent input. In later studies, competition from other short-haul modes (automobile, train, and bus) has been assumed, and the forecast demand has been made a function of access time, speed, and frequency of service, using the abstract mode models of Ref. 13.

2. Passenger-Flow Pattern: *F* Matrix

Unless the system is providing direct nonstop service between all city pairs, d_{AB} will not be the totality of passengers on the route A-B since other "through" passengers will use the route in going X-A-B-Y. For 50 cities, there are 2450 possible nonstop services, but not all of them will generate sufficient demand to warrant direct, or nonstop, service.

It is necessary to have some method of determining the passenger-flow patterns for a given subset of nonstop services. The total passenger flow (nonstop plus through passengers) then determines the daily number of nonstop seats required, or the frequency of daily nonstop services required for a given vehicle seat size and load factor. This process is sometimes called determining the frequency-of-service pattern for the

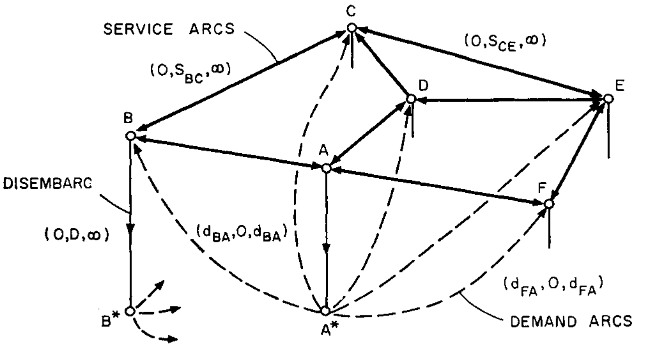

Fig. 1a Network for passenger-flow solution.

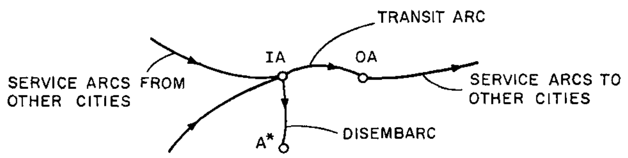

Fig. 1b Modification of city nodes.

system. Lesser routes are dropped to zero frequency in favor of higher frequencies elsewhere.

A multicommodity network flow computation is used to reroute the passengers from X to Y via the shortest routing X-A-B-Y. Initially, an assumption has been made that least distance can be used as a criterion for determining the "shortest" routing. It is probably a good representation of least time when the frequency of service on all routes is fairly high. The model can be extended to be precisely least time when a time-of-day schedule network and a discrete timetable are used as indicated in Ref. 2. It is assumed that a passenger will use the services that have the earliest arrival at Y, and that the indirect routings will not have an effect on the D matrix demands. Neither of these assumptions can be rigorously defended. In the airbus system, it is planned that knowledge of the earliest available arrival at his destination via indirect routings will be available to the passenger through the system's computer information system and that intermediate stops will delay a flight only a few minutes. Both of these factors will tend to make the assumptions more applicable to a VTOL airbus-type of service than to present airline service.

The methods used to determine shortest paths and the passenger F matrix are drawn from network flow theory. A brief explanation of the techniques will be given here.

The computer program used was a modification of an IBM SHARE library coding for the "Out of Kilter" algorithm of Ford and Fulkerson. This algorithm is applicable to large transportation minimum-cost network-flow problems where the number of arcs and nodes can be of the order of 15,000 and 5000, respectively. Solutions give integer values of a circulation flow in a few minutes on a Model 65 IBM 360. To use this technique, it is necessary to construct a network as a model of the passenger-flow problem. Figure 1 shows a simplified network representation of the model.

The subgraph of "city" nodes A, B, C, . . . and "service arcs" AB, BA, AD, DA, . . . represents the geographic or service network of cities A, B, etc., and nonstop services operated between cities AB, BA, etc. If service between cities A and B is to be operated nonstop, then a pair of directed "service arcs" AB and BA exist in the service network. In Fig. 1, this pair of arcs is represented as a solid line with both directions indicated by arrows. Each arc has its cost (c_{ij}) value set to s_{ij}, the distance in miles between the city pair. Every service arc has a lower flow limit, $l_{ij} = 0$, and an upper flow limit $u_{ij} = \infty$.

Thus, there are no capacity constraints on the flow in service arcs, and the value of the flow x_{ij} will represent the number of passengers/day wishing to use this service.

The arcs AA*, BB*, etc. are another subset of arcs called "disembarcs." Nodes A*, B*, C*, . . . are called station nodes. There is one disembarc for each city. Their values of l_{ij} and u_{ij} are 0 and ∞, respectively, for the service arcs, and the cost for these arcs is D, the diameter of the service network. A diameter is the longest track or elementary path in a network, and this value is placed upon these arcs to prevent flows from disembarking at an intermediate station and traveling via demand arcs to their destinations. The value of the flow in a disembarc represents the number of people arriving at that station per day. It will equal the sum of the associated column of the D matrix.

From every station node A*, B*, C*, . . . there is a subset of arcs called "demand" arcs. For example, from node A*, a demand arc exists going to all city nodes B, C, D, ... The cost on these arcs is zero, and both l_{ij} and u_{ij} are set to a value d_{ij} representing the daily demand in passengers/day between city j and city i. These demand arcs insure that a flow x_{ji} equal to the demand d_{ji} exists in the demand arc, and the requirement for a circulation flow necessitates a return flow via the shortest path through the service network. For example, the demand arc A*E in Fig. 1 causes a flow in the service network back to A and A* via either EDA or EFA, whichever is shortest. For an n-city problem, there will be $n \times n - 1$ demand arcs. If the demand is symmetrical, some modifications can be made to avoid repetition of demands.

By a simple trick of splitting the nodes A, B, C, . . . into two parts separated by a "transit" arc, as indicated in Fig. 1b, further information can be obtained. For example, node A becomes two nodes: IA which receives all service arcs from other cities, and OA which starts all service arcs out of city A. The transit arc (IA OA) has a flow that is the number of people passing through the station on their trips to other destinations. For a system serving the 50 stopping points that were selected, such a model was constructed. There are 2450 demand arcs, 50 disembarcs, 50 transit arcs, and 200–500 service arcs. The value of D was set at 1000 miles.

Several solutions were obtained for various demand matrices and different service networks. The service networks were selected on two criteria. The first was geographic, where each station was connected to two of its closest neighbors by "basic" arcs. These basic arcs were members of every service network. The second criterion was either demand d_{ij} or passenger flow x_{ij}. A strategy of adding service arcs of lesser demand d_{ij} was followed in the first few runs. Adding more and more nonstop services diluted the flows to the point where small passenger flows existed on various lesser services. The strategy then became that of dropping service on these low-density routes and requiring these passengers to proceed indirectly via alternate routes that enjoyed higher passenger flows. Basic service arcs were always retained so that no station could be isolated.

Typical results are given in detail in Fig. 2. Entries in the upper half of the matrix are x_{ij} = passengers per day using service ij. Other interesting summary statistics associated with Fig. 2 results are: total passenger (pax)-miles/day, 11.2×10^6; total pax-trips/day, 76,000; pax departures/day, 142,454; av pax trip distance, 148.5 miles; av flight distance, 79.1 miles; and av hops/passenger, 1.9.

3. Frequency Pattern: *N* Matrix

Given the expected number of passengers/day using a given route, an estimate of the desirable number of services or direct flights/day can be obtained by using vehicle seat size and a desirable average load factor established by competitive circumstances or management planning policy. If we let

F MATRIX — DAILY PASSENGERS - BOTH WAYS (above the diagonal)

N MATRIX — DAILY FREQUENCY - ONE WAY (below the diagonal)

	DCA	DUL	BAL	BOS	BOC	MAN	WOR	NBD	PVD	WIL	PHL	TRE	REA	LAN	ALL	HAR	WBA	SPR	PIT	BDL	JFK	LGA	JRB	EWR	BDR
DCA		126																			659	766			
DUL	2																							436	
BAL																									
BOS					1087																700	1135			
BOC				11			419		528												594	2324		830	
MAN																									
WOR					5													530				1860			
NBD									278																
PVD					6			3													627	1533			
WIL											1206											835			
PHL										13				566							968	1798		614	
TRE													90									538	219		
REA												1		1703	1569	288	899					629			
LAN											6		18									755			
ALL													16									751		751	
HAR													3									638		684	
WBA													9									409			
SPR							6												366	585		1531			
PIT																		4							
BDL																		6							
JFK	7			7	6				7		10											1959	182	602	
LGA	8			12	24		19		16	9	18	6	7	8	8	7	5	16			20		1796		1374
JRB												3									2	18		878	
EWR		5			9						7				8	7					7		9		
BDR																						14			
HVN																						20			12
WBY																									
CON								11	6													11			
HFD				7	6				2											12		14		7	
ACY																							7		
BMR			12							7	17			16							6	19		8	
WAS	18	8	3								15										6	10		10	
MIT					9				12		14										10	14			
TBO											8												2		
NYC				21							9				6						3	32	4	14	
ORF																						10			
RIC	7																				5				
PPA	14	17	15	12	7						24	5	12					10			16	29	18	11	
PHF																					8				
YRK														6		7						7			
AVP																	7								
SGC											8											8			
NWK																									9
ISP																									
EQU																						6			
EHM																									
PWM																									
FIT					1	4	8												1						
BTN					2			9																	
LAW				14		1																			

	HVN	WBY	CON	HFD	ACY	BMR	WAS	MIT	TBO	NYC	ORF	RIC	PPA	PHF	YRK	AVP	SGC	NWK	ISP	EQU	EHM	PWM	FIT	BTN	LAW
DCA							1799					620	1374												
DUL							752						1687												
BAL						1181	280						1445												
BOS				667						2064			1111												1301
BOC				583				827					607										65	112	
MAN																							399		49
WOR																							751		
NBD			1083																					854	
PVD			587	178				1172																	
WIL						633																			
PHL						1616	1420	1351	747	899			2373				725								
TRE													423												
REA													1106												
LAN						1552									524										
ALL										593															
HAR															692										
WBA																685									
SPR													936												
PIT																							39		
BDL				1188																					
JFK						570	522	973		259		484	1519	794											
LGA	2000		1012	1345		1886	983	1331		3172	942		2869		604		763			581					
JRB					617				120	340			1714												
EWR				646		715	960			1307			1029												
BDR	1197																	847							
HVN		851	572	1595															622						
WBY	9			161																					
CON	6												758								343				
HFD	16	2											805												
ACY													401												
BMR							1664			763	514	745	1380		720										
WAS						17		509		733	503		1424												
MIT							6				548		1613				261		1683						
TBO										2093			867			824									
NYC						8	8		21				957												
ORF						6	6	6					1158	70											
RIC						8							1094	616											
PPA			8	9	5	14	15	17	9	10	12	11		850			738			575					
PHF											1	7	9												
YRK						8																			
AVP									9																
SGC								3					8					1190							
NWK																	12								
ISP	7							17												288					
EQU													6						3		672				
EHM			4																	7					
PWM																									518
FIT																									
BTN																									
LAW																						6			

Fig. 2 The passenger-flow matrix (*F*) and the frequency pattern matrix (*N*).

n_{ij} be the number of flights/day,

$$n_{ij} = x_{ij}/\langle S \cdot LF \rangle$$

where x_{ij} = daily passengers from i to j—one way; S = vehicle seat size; LF = desired average load factor; and $\langle\ \rangle$ = rounding off to next highest integer.

Initially, it was assumed that there was only one vehicle size (80 passengers); and that for planning purposes, an average system load factor which should be achievable was 60%.

The average load factor of 60% was chosen to allow for monthly and weekly cycles of demand, and to account for the fact that daily demand is a probabilistic variate from day to day. Later studies have allowed load factor to vary throughout the day using the optimal dispatching methods of Refs. 8 and 9 to minimize passenger delay and dispatch costs. As well, the vehicle size has been selected to maximize net return to the operator given the estimated effects of frequency of service on revenue. Various methods of routing two or more fleets in the schedule network similar to the method of Ref. 4 are being studied.

A frequency pattern, or N matrix, corresponding to the F matrix is given in Fig. 2 below the diagonal. It assumes an 80-passenger vehicle at 60% load factor, and is symmetric. The entries are the number of one-way flights/day for each service. It is interesting to note at this point that the N matrix of Fig. 2 has 2996 flights/day, which is greater than any airline schedule in existence today.

4. Timetable Construction

Having determined a frequency pattern for all nonstop services, the next step in constructing a timetable or schedule plan is to assign departure times for each of the n_{ij} services on every route ij. Given a departure time for a flight from i to j and knowledge of the trip duration or block time, the arrival time at j is determined. The set of departure and arrival times, properly ordered for every station, constitutes a timetable describing in explicit detail the tranportation system operations.

A computer program has been written to construct an initial timetable, given as input at this point three sets of data: 1) the N matrix, or frequency pattern giving n_{ij}; 2) the T matrix describing block times for route ij; 3) data describing the daily variation in demand d_{ij} (t), for every city pair.

In the absence of detailed information about daily variations in demand for the hypothetical 1980 airbus system, two demand variations were assumed: a flat distribution from 0600 to 2400 hr, and an extremely peaked distribution descriptive of Eastern Airlines (EAL) shuttle demand on a Friday. It was considered that these were extremes and that the average daily variation would lie somewhere between them.

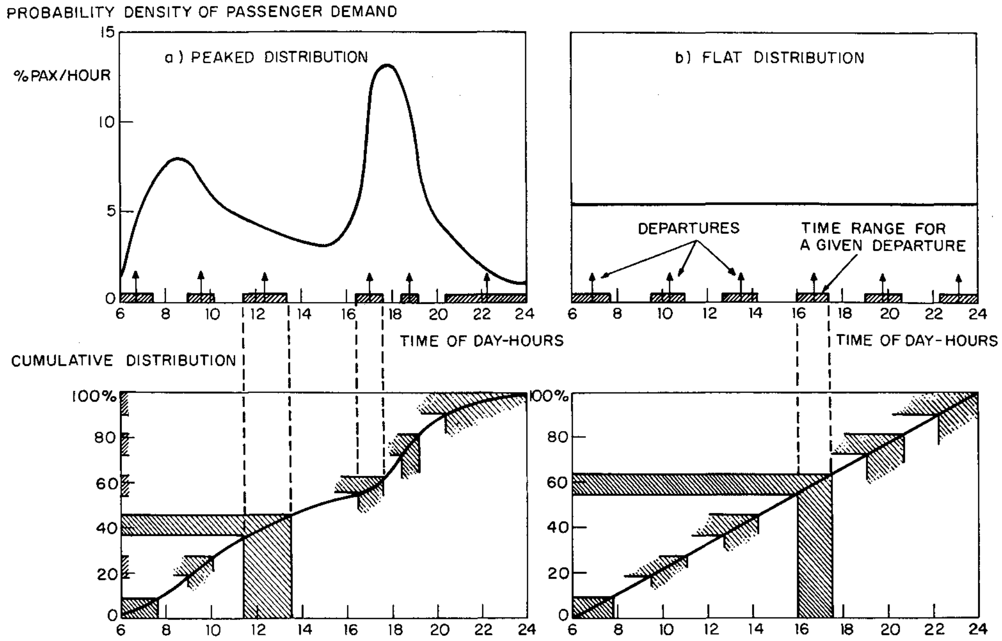

Fig. 3 Method of determining departure times—example for 6 departures/day.

It is possible that the feasibility of a half-day business trip using the airbus system will modify the 9 and 5 peaking patterns typical of present weekday air shuttle patterns.

The initial method of choosing departure times for n_{ij} flights in ij service is explained by Fig. 3. (The optimal dispatching methods now being used do not give very different results. The two distributions of the arrival rate of passengers (or of the probability density of passenger demand) are shown in the upper portion of Fig. 3. Below them are the cumulative probabilities of demand obtained by integrating the probability density distributions or arrival rates. It goes from 0 to 100%, and represents the cumulative number of passengers arriving during the day. The uniform arrival rate gives a straight line from 0600 to 2400, whereas the peaking distribution shows much steeper slopes during the peak periods.

The rationale first used to select times was to divide the daily load on a given route equally among n_{ij} departures by dividing the vertical axis of the cumulative distribution into n_{ij} equal segments. The departure times could then be found by reading the corresponding time from the horizontal axis. However, because of the optimization process described in the next section, this was changed such that a range of times was selected for each departure in a similar manner. Thus, the vertical axis was divided into $2n_{ij}-1$ parts so that n_{ij} departure ranges could be selected. The departure time was placed in the center of each range to form an initial timetable.

This is shown in Fig. 3 for both distributions when $n_{ij} = 6$. The vertical axis is divided into 13 segments, and the departure ranges are shown by the shaded bands. For the uniform or flat distribution, the departures are equally spaced. For the peaked distribution, the departure times tend to be bunched at the peak hours, when their ranges are also much reduced. There is always a gap between departure ranges such that two successive departures for the same destination will always be separated even if they are moved to the closest end points of their ranges.

There has been no consideration of continuing through flights at this point. They would be constructed after seeing the connectivity of the flight hops after the optimization process of the next section. Similarly, there has been no consideration of interconnections between other modes of transportation. The possibility of competition (a critical factor in choosing times for airline schedules) has not been considered here. All these considerations would be introduced at a later stage if pertinent information were available. Notice that the departure times are chosen from the same daily variation for all routes representative of O and D demand, not indirect demand through a station.

5. Optimization of Vehicle Utilization

A range of departure times for each service was chosen so that departure times could be varied to allow better connections for vehicles and passengers. In this way, vehicle utilization could be improved. This has a strong effect on direct operating cost, since depreciation costs for a typical 3-4 million dollar airbus vehicle in short-haul service are about 30% of the direct operating cost (DOC). Maximizing utilization is equivalent to minimizing either ground time or the number of aircraft in the fleet required for a given timetable. This may be seen from the following.

For a given timetable of services, the total amount of block time is fixed. Let this be called BT, for the daily number of hours flown in the schedule. Average vehicle utilization U in terms of average hours/day per vehicle, given that fleet size is NF, is simply $U = BT/NF$. Therefore, since BT is constant, U is maximized when NF is minimized.

However, if there are NF aircraft in the fleet, there are $24\cdot(NF)$ available fleet hours/day. We can account for all of these fleet hours by classifying the ground time into two parts: ST, a stopping time required for load-unload-refuel in transiting a station; WT, a ground-wait time where aircraft could be available for service if desired. Since BT and ST are constant for a given set of services to be operated, we can write

$$24\cdot(NF) = BT + ST + WT$$

but $BT + ST = K = \text{const}$

$$\therefore 24\cdot NF = K + WT$$

From this relationship, we see that if NF remains constant, WT must be fixed. But NF must be an integer number, and can only be reduced in steps of unity. Each unit step will reduce total fleet time by 24 hr, and the reduction must come in WT. Therefore, total WT for any schedule can only be reduced in increments of 24 hr and corresponds directly to the elimination of one aircraft from the required fleet. It is possible, therefore, to concentrate on the elimination of aircraft in order to optimize fleet utilization.

In this section, a heuristic algorithm will be described which minimizes NF, given a schedule and a description of the allowable ranges for every departure time in that schedule. It does not achieve a true optimum. It is one of three developed at MIT in the last year which have the simple capability of reducing NF with varying degrees of success. Obtaining the true optimum for such sequencing problems seems to be beyond the state-of-the-art for operations research at the present time. The use of similar methods or simulation on "job-shop" problems is typical for such sequencing problems.

A basic assumption implicit in this statement of this problem is that services can be varied within some range without any change in the amount of revenue or traffic associated with the flight. In airline practice, where competition may exist, this range can be very small. However, in other cases, the airline marketing analysis often associates a broad range of times with the service. The ranges are chosen rather arbitrarily in this study because of the lack of any detailed data. The algorithm will accept any well-defined range for every service. Departure times can be fixed by having the upper and lower limits of the range coincide. Some latitude in departure times is necessary of course for the optimization to be able to operate.

For every station, the timetable describes a list of time-ordered events of two types: first, a "ready time" event when an aircraft arriving from another station becomes ready or available for service after unloading and refueling; second, a departure event for services to other stations. Table 1 shows such a typical event sequence E. If we define NA to be the number of aircraft on the ground after each event, the NA sequence consists of numbers that differ by unity. For a departure, 1 is subtracted from the previous NA and for an arrival "ready," 1 is added to the present event's NA. We may start the NA sequence with any large number NAC, which represents the number of vehicles which will "overnight" at the station. Table 1 uses $NAC = 100$ in starting the sequence in the column NA_1.

If we find the smallest member in the NA_1 sequence, and subtract it from every number of the sequence, we get the NA_2 sequence that will have a number of zeros (at least one) appearing somewhere in the sequence. This sequence represents the minimum number of vehicles required to carry out the timetable at this station. The total minimum fleet NF can be counted by adding NAC for every station, i.e., the total number of aircraft overnighting at all stations. This assumes that there is some period during the night at which the total fleet is on the ground. This is usually possible for short-haul passenger transport schedules.

Table 1 Method of counting vehicles at a station

Time	E	NA_1[b]	NA_2
		Put $NAC = 100$	Put $NAC = 3$
0715	D[a]	99	2
0730	D	98	1
0800	D	97*	0
0855	R[c]	98	1
0930	R	99	2
1100	D	98	1
1215	R	99	2
1400	D	98	1
1705	R	99	2
1730	D	98	1
1920	D	97*	0
2040	R	98	1
2100	D	97*	0
2245	R	98	1
2305	R	99	2
2330	R	100	3
Number of aircraft overnight		$NAC = 100$	$NAC = 3$
Smallest number in NA sequence		$= 97$	$= 0$

[a] D = Departure.
[b] NA = Number of aircraft at station after each event.
[c] R = Arrival ready.

Table 2 Example of reducing NAC and NF

	Initial Order of Events at j		Revised Order of Events at j		
	E	$NA(j)$	E^*	$NA^*(j)$	$NA^*(j)-1$
	D	1	D	1	0
Previous Station i					
•	R	2	R	2	1
•					
•	D	1	D	1	0
D			R	2	1
D (No change in NA(i))	D	0	D	1	0
D					
•	R	1			
•					
•	R	2	R	2	1
			Subtract unity		
NAC(i) = constant	NAC(j) = 2				NAC*(j) = 1
∴ NF* = NF − 1					

The connections or "turns" which the vehicles on the ground make between incoming and outgoing services is not explicitly stated. If there is only one aircraft on the ground before a departure, then it must be used on the departure service. However, if there are two or more, any one of them can be used since they are all ready for service.

If we adopt a strategy for connections of "last in-first out," then we can show that the NA_2 sequence is truly minimal. For if there is one (or more) aircraft on the ground at all times, it is never used in any service and is unnecessary (except perhaps as a spare or "cover" aircraft for schedule reliability). To use the last vehicle, a zero must appear after a departure at least once in the minimal NA sequence. Of course, NA cannot contain a negative number since it would represent a negative number of vehicles on the ground. This counting logic is well known to schedulers and even has been discovered by more sophisticated methods of operations research!

If we are given a timetable and a corresponding set of minimal NA sequences, we may be able to reduce NAC for any station j by interchanging departure and arrival events such as to increase the zero values in the NA (j) sequence. If it is possible to increase all the zeros in the NA (j) sequence to unity, then we have a new sequence of events, E^*, where the NA^* (j) sequence is no longer minimal. The new minimal NA^* (j) sequence is obtained by subtracting unity from every member of the sequence. The last member of the sequence is NAC^* (j), which is thereby reduced by 1. Providing the interchange of events a station j did not increase NAC at the previous stations (i) and downstream stations (k), then the fleet size NF will have been decreased by unity.

An example of this logic can be given with the aid of Table 2. For the original sequence of events at station j, a zero appears after the third departure. It is possible to change this zero to unity in two ways: 1) move the corresponding departure after any of the following arrivals and 2) move a later arrival ahead of the zero departure—provided the arrival remains within the range of times associated with its flight.

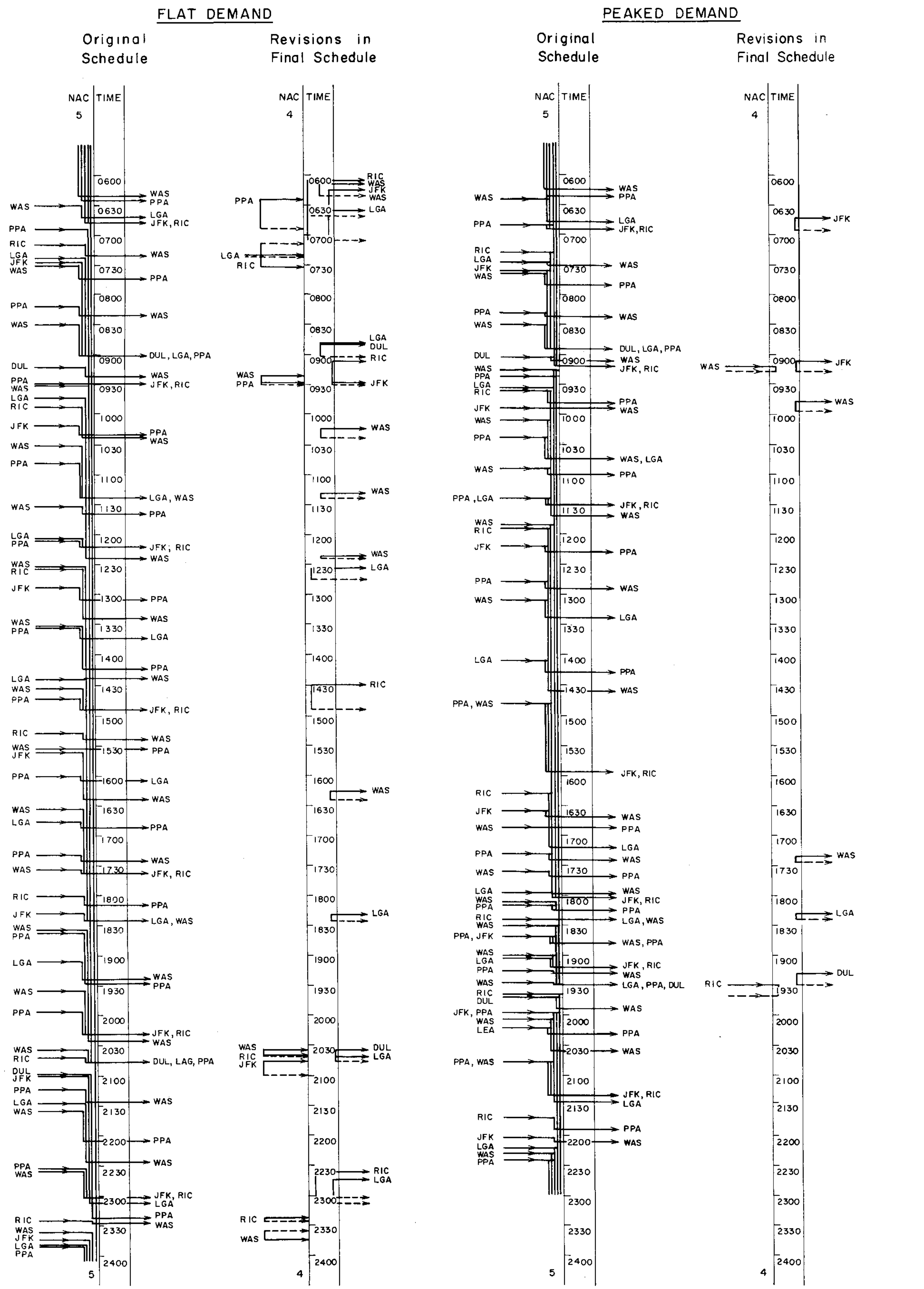

Fig. 4 Typical station schedules—Washington National with 112 operations/day to and from New York JFK, New York LGA, Washington WAS, Richmond RIC, and Philadelphia downtown, PPA.

In the example, the next arrival has been moved ahead of the zero departure, and there has been no change in the *NA* (*i*) sequence, and therefore *NAC* (*i*) remains constant. The revised *NA** (*j*) sequence is no longer minimal. Unity can be subtracted, giving a new minimal *NA** (*j*) sequence of 01010 . . . , and reducing *NAC** (*j*) to unity. *NF** is also reduced by unity.

The same sequence of events could have obtained by moving the zero departure after the next arrival, provided the move is within the departure range, and that any change at the corresponding arrival station *k* did not increase *NAC* (*k*). Although the sequence is the same, the times associated with the departure and arrival are different in the two alternatives.

One further observation is that any change made to reduce *NAC* (*j*) will connect two flight segments into a continuing flight. The input of services in the study has been individual services consisting of one flight segment, and the segments are only definitely connected at the time when such a change is made. If the input was to be flights of more than one segment, then the connections could be restricted between the segments and the turns made only at the flight termination, i.e., a flight ABCD can be treated as a flight AD with appropriate times, and the optimization process is similar.

There are two distinct timetables associated with a peaking of services to match demand and a flat distribution of services when load factors are allowed to vary. Table 3 summarizes the pertinent quantities for timetables both before and after the optimization of the schedule. The improvement is quite marked (utilizations are roughly doubled) because the initial choice of times for services did not take into account the connectivity between flights. It shows the sensitivity of the utilization to such considerations, and indicates that dynamic scheduling where passenger demand alone determines service will have poor vehicle utilization. A similar algorithm applied to a real airline schedule assuming $\pm\frac{1}{2}$ hr departure ranges gave only a 10% improvement, and times typically had to be changed for seven different flights over four stations to get rid of just one airplane. An airline scheduler would have already tightened the schedule by making good "turns" except at those places where slack was intentionally introduced for schedule reliability, etc.

The effect of peaking is quite marked especially when utilization has been optimized. Daily utilization of 6.75 hr/day drops to 5.00 hr/day for the case where schedules are bunched following the EAL shuttle demand distribution. When the utilization is low, there are apparently sufficient slack airplanes in the schedule to dampen the effect of peak service requirements.

The resulting utilization shows the effect of time spent on the ground awaiting a suitable departure time. For example, the average vehicle spends 6.75 hr/day in block time, for the flat demand schedule. It makes about 27 trips/day and has an assumed total stop time for load-unload of about 2.7 hr/day. The remainder of an 18 hr useful airline day, or 8.55 hr/day, gives 8.55/27 = 19 min as the average time spent waiting for a suitable departure time. Utilization can be increased at the expense of load factor by departing early at less suitable times. The tradeoff is normally not acceptable economically. A detailed econometric model of the airline market and schedule is required to ascertain the marginal revenues and costs involved in such a tradeoff.

6. Resulting Schedules

The schedules which have been constructed and optimized are too large to be completely shown. Figure 4, describing the scheduled operations at Washington National Airport, gives some indication of the size and detail of these schedules. It also shows the changes made to the initial schedule by the process that improves vehicle utilization. As indicated, relatively few flights are changed, and the time changes are of the order of 6 min. The largest time change was 13.2 min.

Table 3 Summary of vehicle utilization from timetable

	Peaked schedule		Flat schedule	
	Initial	Final	Initial	Final
Fleet size, *NF*	251	164	238	121
Utilization, hr/yr	1190	1820	1249	2460
Utilization, hr/day	3.26	5.00	3.42	6.75
Vehicle trips/day	13	20	14	27

Washington National, with 112 operations/day, was the seventeenth busiest station in the resulting schedule. The busiest station was Laguardia, with almost 800 VTOL operations/day and a peak-hour requirement of 87 operations. This schedule result would indicate that another VTOL site should be chosen in mid-Manhattan, and similar considerations made for a second downtown Philadelphia site (646 operations/day). Such results are directly dependent on the forecast demand inputs, which may be described as speculative at this time.

Another conclusion from the schedule result is that the average vehicle hop distance is surprisingly low at less than 80 miles. This information is an input to vehicle design and operation, and is of importance in selecting the most economic type of VTOL vehicle for the airbus system. This result depends more directly upon the size and extent of the route network.

Similarly, the schedule frequencies in various low-density markets seem too low for these very short-haul services. It is necessary to provide much higher frequencies, and a smaller vehicle than the 80-passenger size assumed seems to be indicated.

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